

# Non-equilibrium symmetry restoration beyond one loop

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We calculate the strength of symmetry restoration effects in highly non-equilibrium states which can arise, for example, during preheating after inflation. We show that in certain parameter range the one-loop results are unstable, requiring summation of multiloop diagrams. We solve this problem for the  $O(N)$  model in the large  $N$ -limit and show that the symmetry restoration may be less effective than what predicted by the one-loop estimate.

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It has been recently realized [1] that the process of reheating after the inflationary era [2] may consist of different epochs.

In the first stage, the effects of stimulated decays and annihilation may lead to an extremely effective explosive particle production. The energy is released in the form of inflaton decay products, whose occupation numbers are extremely large, and have energies much smaller than the temperature that would have been obtained by an instantaneous conversion of the inflaton energy density into equilibrated radiation.

Since it requires several scattering times for the low-energy decay products to form a thermal distribution, it is rather reasonable to consider the period in which most of the energy density of the Universe was in the form of non-thermal quanta produced by inflaton decay as a separate cosmological era. Following [1] we refer to the final stages of parametric resonance as the *preheating* epoch to distinguish it from the subsequent stages of particle decay and semiclassical thermalization [3] when the inflaton decay can be still efficient, but amplitudes of fluctuating fields became gradually smaller because of Universe expansion.

Several aspects of the theory of explosive reheating have been studied in the case of slow-roll inflation [4] and first-order inflation [5]. In particular, the phenomenon of symmetry restoration during the preheating era has been investigated recently by Kofman, Linde, and Starobinski [6] and by Tkachev [7] in the framework of typical chaotic inflationary models. It was shown that nonthermal symmetry restoration processes during the nonequilibrium stage of preheating may be very efficient with important implications for models containing relatively low scales like the invisible axion model or supersymmetry with Polonyi fields, and may be for Grand Unified Theories (GUT). For example, if a GUT symmetry is restored during the preheating epoch, the subsequent symmetry breaking phase transition will reintroduce the problems of monopoles [8] or domain walls [9]. In any case, if one asks whether particular high-energy symmetries are restored after

inflation or not, one has to study the non-equilibrium preheating epoch.

We consider a model with the following potential

$$V_0(\phi, \eta) = \lambda\phi^4 + \frac{\alpha}{4!} (\eta^2 - \eta_0^2)^2 + \frac{g}{2} \phi^2 \eta^2. \quad (1)$$

The field  $\phi$  corresponds to the inflaton. While inflaton self-coupling is restricted to be  $\lambda \approx 10^{-13}$ , there is no such severe restrictions on other couplings. Among products of the inflaton decay there can be different species, but for simplicity we assume here that quanta of the field  $\eta$  constitute the major channel of its decay, which means in particular that  $g \gg \lambda$ . We assume also that  $\eta$  transforms as a vector under the action of  $O(N)$ , *i.e.*  $\eta^2 = \eta_a \eta^a$  with  $a = 1, \dots, N$ .

In previous papers [1],[6],[7] the effects of the coupling  $\alpha$  was neglected, which corresponds, in fact, to the one loop approximation. The importance of multiloop correction was noticed in Ref. [7]. We shall show here that they change the results dramatically when  $\alpha \gg g$  and the one-loop approximation actually breaks down in this region.

We do not consider an implications of non-zero  $\alpha$  on parametric resonance itself, but consider the effects of symmetry restoration only. We assume that at the end of the resonance stage the energy density which builds up in the fluctuations of field  $\eta$ , and which we denote as  $\rho_\eta$ , is some fraction of inflaton energy density. Simple assumption would be that this fraction equals to one half [1]. Numerical integration of Ref. [3] for the case of simplest  $\lambda\phi^4$  model confirms that the resonance ends when half of the energy is in fluctuations, however it might not be true for  $g \gg \lambda$ . Moreover, because of the expansion of the Universe during preheating,  $\rho_\eta$  is many orders of magnitude smaller than the initial inflaton energy density. Since this is model dependent, at present we keep  $\rho_\eta$  as a free parameter.

We can parametrize the distribution function of the created quanta at preheating as  $f \equiv f(k/k_*)$  where  $k$  is particle momenta. What is important here is the smallness of

typical particle momenta,  $k_*$ , with respect to the temperature that one would get in the case of instantaneous reheating neglecting the expansion of the Universe. In the extreme case when all inflaton couplings are not very different,  $k_*$  is of order of inflaton mass,  $k_* \sim 10^{13}$  GeV. For definiteness, let us assume that the distribution function is of the form

$$f(k) = A \delta(|\mathbf{k}| - |\mathbf{k}_*|). \quad (2)$$

Parametrically, and that is what we are interested in here, Eq. (2) will reproduce the correct answer. Moreover, the distribution is really peaked at the end of the resonance stage in reasonably wide range of model parameters.

The constant  $A$  (or its initial value in our problem) can be fixed by computing the energy density of  $\eta$  particles and setting it equal to  $\rho_\eta$ . Notice that, since the order parameter changes in time, *e.g.* during the process of symmetry restoration, the energy of particles coupled to it changes too, but three-momenta keep constant in an homogeneous background. Moreover, in this case the effective potential simply coincides with the energy density, which simplifies the calculations. To answer the question whether the symmetry tends to be restored or not, it is sufficient to consider homogeneous background only.

As opposed to large-angle scattering processes, forward-scattering processes do not alter the distribution function of the particles traversing a gas of quanta, but simply modify the dispersion relation. This remains true also in the case of a nonequilibrium system. Forward scattering is manifest, for example, as ensemble and scalar background corrections to the particle masses. Since the forward scattering rate is usually larger than the large-angle scattering rate responsible for establishing a thermal distribution, the nonequilibrium ensemble and scalar background corrections are present even before the initial distribution function relaxes to its thermal value. These considerations allow

us to impose  $k^0 = \sqrt{|\mathbf{k}|^2 + m_\eta^2(\bar{\eta})}$  as the dispersion relation for the particles created by the parametric resonance, where  $\bar{\eta}$  is the order parameter, with  $\bar{\eta} = \eta_0$  in vacuum, and  $m_\eta^2(\bar{\eta})\delta_{ab} = \alpha [(\bar{\eta}^2 - \eta_0^2)\delta_{ab} + 2\bar{\eta}_a\bar{\eta}_b] / 3!$ .

Let us first consider the case  $N = 1$ . We can not use the imaginary-time formalism to determine the effective potential for the scalar field  $\bar{\eta}$  during the nonequilibrium preheating period since in the nonequilibrium case there is no relation between the density matrix of the system and the time evolution operator, which is of essential importance in the formalism. There is, however, the real-time formalism of Thermo Field Dynamics (TFD), which suites our purposes [10]. This approach leads to a  $2 \times 2$  matrix structure for the free propagator (only the (11)-component is physical)

$$\begin{pmatrix} D_{11}(K) & D_{12}(K) \\ D_{21}(K) & D_{22}(K) \end{pmatrix} = \begin{pmatrix} \Delta(K) & 0 \\ 0 & \Delta^*(K) \end{pmatrix} + \begin{pmatrix} f(k) & \theta(k_0) + f(k) \\ \theta(-k_0) + f(k) & f(k) \end{pmatrix} \times 2\pi\delta[K^2 - m_\eta^2(\bar{\eta})], \quad (3)$$

with the usual vacuum Feynman propagator

$$\Delta(K) = \frac{i}{K^2 - m_\eta^2(\bar{\eta}) + i\epsilon}. \quad (4)$$

The one-loop effective potential  $V_1(\bar{\eta})$  can be easily obtained solving the equation

$$\frac{dV_1}{dm_\eta^2} = \mathcal{T}_1 = \frac{1}{2} \int \frac{d^4K}{(2\pi)^4} D_{11}(K) \equiv \frac{1}{2} (I_0 + I_f), \quad (5)$$

where  $\mathcal{T}_1$  is the one-loop tadpole diagram which receives contribution only from the diagram in Fig. 1 (since the final external leg is fixed to be that of the physical field) and we have separated the usual zero-temperature contribution from the one given by the gas of  $\eta$ -quanta with distribution function  $f$ , Eq. (2). One then obtains

$$V_1(\bar{\eta}) = V_1^{(0)} + V_1^{(f)} = V_1^{(0)} + \frac{A |\mathbf{k}_*|^2}{4\pi^2} \sqrt{|\mathbf{k}_*|^2 + m_\eta^2(\bar{\eta})}, \quad (6)$$

where  $V_1^{(0)}$  is the one-loop zero-temperature effective potential. With the distribution function Eq. (2), the particle number density is

$$n = \frac{1}{(2\pi)^3} \int d^3k f(k) = \frac{A |\mathbf{k}_*|^2}{2\pi^2}, \quad (7)$$

and the particle-dependent part of the effective potential reads  $V_1^{(f)} = n\sqrt{|\mathbf{k}_*|^2 + m_\eta^2(\bar{\eta})}/2$ . We would like to mention for the future use that Eq. (7) is valid only in the one-loop approximation. Expanding  $V_1^{(f)}$  around  $\bar{\eta} = 0$ , we obtain that the coefficient of  $\bar{\eta}^2$ -term in the potential (or the effective mass of the field) is changed by interaction with the medium by an amount  $\alpha n/|\mathbf{k}_*| \sim \alpha \rho_\eta/|\mathbf{k}_*|^2$  [6], [7]. The presence of nonthermalized  $\eta$ -quanta may lead to symmetry restoration if  $\rho_\eta/|\mathbf{k}_*|^2 \gtrsim \eta_0^2$ .

Our main concern in this Letter is on what happens when multi-loop corrections are included in the computation of the nonequilibrium effective potential. We will show that in certain range of parameters the effective potential gets large contributions at the two-loop level and higher orders of perturbation theory, even larger than the one-loop contribution. Perturbation theory is then lost unless a proper resummation can be done. We give an example in which the resummation can be performed exactly.

Let us first consider the theory described by the potential in Eq. (1). By applying the rules of TFD we obtain the two-loop tadpole diagrams as drawn in Fig. 2 (plus counter-terms arising at one-loop). Underlined diagrams are identically zero since they contain  $\delta(K^2 - m^2)\delta((K - P)^2 - m^2)\delta(P^2 - m^2) = 0$ .

We first consider the simplest case of diagram 2a

$$\frac{dV_{(2a)}}{dm_\eta^2} = \frac{i\alpha}{4} \int \frac{d^4K}{(2\pi)^4} \int \frac{d^4P}{(2\pi)^4} \left[ D_{11}^2(P) D_{11}(K) - D_{12}(P) D_{21}(P) D_{22}(K) \right]. \quad (8)$$

Observing that the tadpole does not have any imaginary part and that  $\text{Re } D_{11}(K) = \text{Re } D_{22}(K)$ , one can factorize the  $K$ -integral out and, using the mass-derivative formula [11], obtain

$$\begin{aligned} \frac{dV_{(2a)}}{dm_\eta^2} &= \frac{i\alpha}{4} \int \frac{d^4K}{(2\pi)^4} \text{Re } D_{11}(K) \frac{d^4P}{(2\pi)^4} \left[ \Delta^2(P) + f \left( \Delta^2(P) - (\Delta^*(P))^2 \right) \right] \\ &= \frac{i\alpha}{4} \int \frac{d^4K}{(2\pi)^4} \text{Re } D_{11}(K) \left[ \int \frac{d^4P}{(2\pi)^4} \Delta^2(P) - i \frac{\partial I_f}{\partial m_\eta^2} \right] \\ &= \frac{i\alpha}{4} \text{Re} (I_0 + I_f) \left( J_0 - i \frac{\partial I_f}{\partial m_\eta^2} \right), \end{aligned} \quad (9)$$

where we have defined  $J_0 = (2\pi)^{-4} \int d^4P \Delta^2(P)$ . Eq. (9) shows the absence of pinch singularities, or  $\Delta(P)\Delta^*(P)$  terms in the final result. The cancellation procedure works essentially in the same way as in the equilibrium case and is totally independent of the distribution  $f$  [12]. After renormalization, the term proportional to  $I_f J_0$  gives a contribution  $\sim \alpha I_f \ln m_\eta^2$  which is suppressed by  $\mathcal{O}(\alpha)$  with respect to the one-loop result. One should not claim victory too soon, though. Let us extract the  $f$ -dependent part only: its contribution goes like  $\alpha I_f \left( \partial I_f / \partial m_\eta^2 \right)$ . Expanding such an expression around  $\bar{\eta} = 0$ , we see that the contribution of diagrams of Fig. 2a is of order of  $(\alpha \rho_\eta / |\mathbf{k}_*|^4)$  with respect to the one-loop result. Since  $\alpha$  may be  $\sim 1$  and  $|\mathbf{k}_*|^4 / \rho_\eta \ll 1$  at the preheating era, we discover that the contribution from the two-loop tadpole diagrams is much larger than the one-loop result by several orders of magnitude and perturbation theory is lost!

Things get even worse when we consider  $p$ -loop diagrams. The tadpole diagrams shown in Fig. 3 ( $p \geq 2$ ) contribute

$$\frac{dV_{(3)}}{dm_\eta^2} = \frac{1}{2} \left( \frac{i\alpha}{2} \right)^{p-1} \text{Re} (I_0 + I_f) \left( J_0 - i \frac{\partial I_f}{\partial m_\eta^2} \right)^{p-1}. \quad (10)$$

All the terms which contain a vacuum contribution are ultraviolet divergent and by cancellation with the counter-terms we are left with only the  $f$ -dependent part which has the following behaviour (with respect to the one-loop result)

$$\left( \alpha \frac{\partial I_f}{\partial m_\eta^2} \right)_{\phi=0}^{p-1} \sim \left( \frac{\alpha \rho_\eta}{|\mathbf{k}_*|^4} \right)^{\frac{p-1}{2}} \gg 1. \quad (11)$$

It is possible to add the tadpoles in a different way. Let us consider the  $p$ -loop contributions shown in Fig. 4, the so-called daisy diagrams. Using again the mass-derivative formula, we obtain the expression for a product of  $p \geq 2$  successive propagators

$$\frac{dV_{(4)}}{dm_\eta^2} = \frac{1}{2} \left( \frac{\alpha}{2} \right)^p (I_0 + I_f)^p \frac{1}{p!} \left( \frac{\partial}{\partial m_\eta^2} \right)^p (I_0 + I_f). \quad (12)$$

This expression has also to be renormalized. For  $p \geq 2$  the ultraviolet singularities are

resummed in  $I_0$  and are cancelled by adding a series of counter-terms arising at one-loop.

The result is

$$\frac{dV_{(4)}}{dm_\eta^2} + \text{C.T.} = \frac{1}{2} \left(\frac{\alpha}{2}\right)^p I_f^N \frac{1}{p!} \left(\frac{\partial}{\partial m_\eta^2}\right)^p (I_0 + I_f). \quad (13)$$

It is clear that successive derivatives with respect to the squared mass give increasing inverse powers of the squared mass. The derivatives of  $I_0$  give a subleading contribution with respect to the  $f$ -dependent part. Extracting the  $f$ -dependent part and expanding such it around  $\bar{\eta} = 0$ , we discover that the contribution of the  $p$ -loop daisy diagrams is again of order of  $(\alpha \rho_\eta / |\mathbf{k}_*|^4)^p \gg 1$  with respect to the one-loop result.

The discussion above shows that in order to obtain a more accurate information about the issue of nonthermal symmetry restoration one must study an infinite series of diagrams in perturbation theory. This is exactly analogous to what happens in a simple  $\lambda\varphi^4$  theory in equilibrium at finite temperature where the leading contributions to the effective potential in the infrared region come from the daisy and superdaisy multiloop graphs [13].

To deal with this problem we need a self-consistent loop expansion of the effective potential in terms of the *full* propagator. Such a technique was developed some time ago by Cornwall, Jackiw and Tomboulis (CJT) in their effective action formalism for composite operators[14] and may be also applied to nonequilibrium phenomena [15]. One considers a generalization  $\Gamma[\bar{\eta}, G]$  of the usual effective action, which depends not only on  $\bar{\eta}(x)$ , but also on  $G(x, y)$ , a possible expectation value of the time-ordered product  $\langle T \eta(x) \eta(y) \rangle$ . The physical solutions satisfy the stationary requirements

$$\begin{aligned} \frac{\delta\Gamma[\bar{\eta}, G]}{\delta\bar{\eta}(x)} &= 0, \\ \frac{\delta\Gamma[\bar{\eta}, G]}{\delta G(x, y)} &= 0. \end{aligned} \quad (14)$$

The conventional effective action  $\Gamma[\bar{\eta}]$  is given by  $\Gamma[\bar{\eta}, G]$  at the solution  $G_0(\bar{\eta})$  of Eq.



(14). In this formalism it is possible to sum a large class of ordinary perturbation-series diagrams that contribute to the effective action  $\Gamma[\bar{\eta}]$ , and the gap equation which determines the form of the full propagator is obtained by a variational technique.

We now apply the CJT formalism in the limit of large  $N$  when the next-to-leading terms can be exactly summed. In each order we keep only the term dominant in  $N$  for large values of  $N$ . This allows us to resum exactly the series of the leading multiloop diagrams and to solve the gap equation for the full propagator without any approximation.

In order to obtain a series expansion of the effective action, one introduces the functional operator

$$D_{ab}^{-1}(\bar{\eta}, x, y) = \frac{\delta^2 I}{\delta \bar{\eta}_a(x) \delta \bar{\eta}_b(y)}, \quad (15)$$

where  $I$  is the classical action. The required series obtained by CJT is then [14]

$$\Gamma[\bar{\eta}, G] = I(\bar{\eta}) + \frac{1}{2} \text{Tr} \ln D_0 G^{-1} + \frac{1}{2} \text{Tr} [D^{-1} G - 1] + \Gamma_2[\bar{\eta}, G], \quad (16)$$

where  $D_0^{-1} = -(\partial_\mu \partial^\mu + m^2) \delta_{ab} \delta^4(x, y)$ , and  $\Gamma_2[\bar{\eta}, G]$  is a sum of all two-particle-irreducible vacuum graphs in the theory with vertices defined by the classical action with shifted fields  $I[\bar{\eta} + \eta]$  and propagators set equal to  $G(x, y)$ .

Previous calculations show that among the multiloop graphs contributing to the effective potential in the  $O(N)$ -theory, only the daisy and superdaisy diagrams survive in the limit of large  $N$  [16]. This enables us to consider in  $\Gamma_2[\bar{\eta}, G]$  only the graph of  $\mathcal{O}(\alpha)$  given in Fig. 5. This is the Hartree-Fock approximation which is known to be exact in the many-body version of our large  $N$  limit.

It is more convenient to concentrate on the effective masses rather than on the effective potential. By stationarizing the effective action  $\Gamma[\bar{\eta}, G]$  with respect to  $G_{ab}$ , we

obtain the gap equation

$$G_{ab}^{-1}(x, y) = D_{ab}^{-1}(x, y) + \frac{\alpha}{6} [\delta_{ab} G_{cc}(x, x) + 2 G_{ab}(x, x)] \delta^4(x, y). \quad (17)$$

This equation is exact in the limit of large  $N$  and contains all the informations about the dominant  $N$ -contributions to the full propagator. Indeed, the exact Schwinger-Dyson equation reduces to Eq. (17) for large  $N$ . We Fourier-transform Eq. (17) and take  $\bar{\eta}_a = 0$ . The gap equation then reads [14]

$$M^2 = m^2 + \frac{\alpha}{6} I_f(M^2) = m^2 + \frac{\alpha N A}{24\pi^2} \frac{|\mathbf{k}_*|^2}{\sqrt{|\mathbf{k}_*|^2 + M^2}}. \quad (18)$$

We see that the one-loop results are stable when  $|\mathbf{k}_*|^2 \gg M^2$ , which translates to the condition  $\rho_\eta \ll |\mathbf{k}_*|^4/\alpha$ , where now  $\rho_\eta$  indicates the total energy of the noninteracting gas (the sum over all  $\eta_a$ ). In the opposite case the gap equation is approximately solved by  $M^3 \simeq \alpha N A |\mathbf{k}_*|^2$ . Using  $\rho_\eta \sim \alpha N^2 A^2 |\mathbf{k}_*|^4/M^2$  we find

$$M^2 \simeq \sqrt{\alpha \rho_\eta}. \quad (19)$$

In other words, the strength of symmetry restoration measured in terms of the effective temperature  $T_{\text{eff}}^2/12 \equiv < (\eta - \bar{\eta})^2 >$  is given by  $T_{\text{eff}}^2/12 \approx M^2/\alpha \approx \sqrt{\rho_\eta/\alpha}$ . This result and Eq. (19) are intuitively understandable. Indeed, in this regime the contribution to the energy from self-coupling is important and  $\rho_\eta$  is saturated by the self-interaction term in Eq. (1). This can be obtained in Hartree-Fock approximation directly applied to equations of motion for the  $\eta$ -field (in the large  $N$ -limit Hartree-Fock approximation becomes exact).

To summarize. In the limit  $|\mathbf{k}_*|^4 \gg \alpha \rho_\eta$  the one-loop results are stable and  $T_{\text{eff}}^2 \sim \rho_\eta/|\mathbf{k}_*|^2$ . Note that our parameters,  $|\mathbf{k}_*|$  and  $\rho_\eta$  are outcome of the stage of resonant decay and depend on the couplings  $g$  and  $\lambda$  in Eq. (1). For example, we can expect that  $|\mathbf{k}_*|^2 \sim \sqrt{g\lambda} M_{\text{Pl}}^2$  [1]. In the opposite limit,  $|\mathbf{k}_*|^4 \ll \alpha \rho_\eta$ , we have found  $T_{\text{eff}}^2 \sim$

$\sqrt{\rho_\eta/\alpha}$ . This is smaller than the one-loop result by a factor  $|\mathbf{k}_*|^2/\sqrt{\alpha\rho_\eta}$ . This result remains valid when the inflaton decay products  $\eta$  and the order parameter in question correspond to different fields. The strength of the symmetry restoration in a highly non-equilibrium state cannot be traced by the one-loop result in the case of sufficiently strong interaction of inflaton decay products. This can have important consequences on the issue of symmetry restoration of various symmetries during the preheating era.

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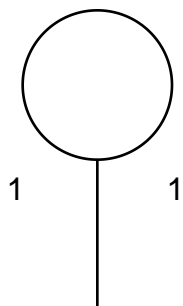


Figure 1

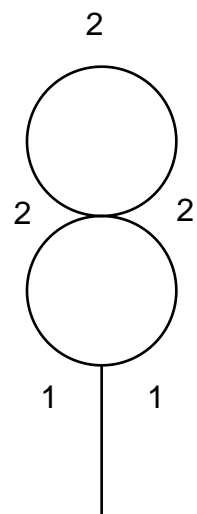
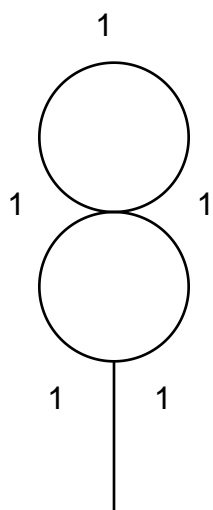


Figure 2 a

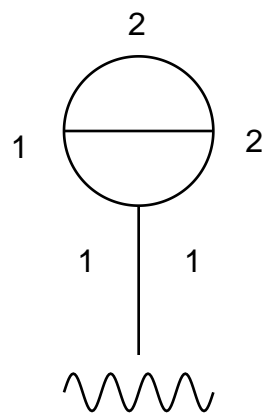
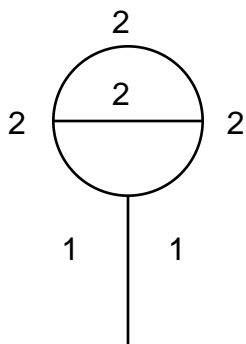
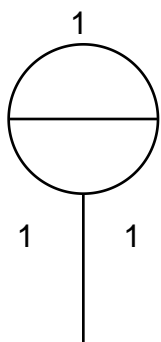


Figure 2 b

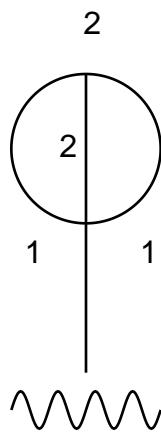
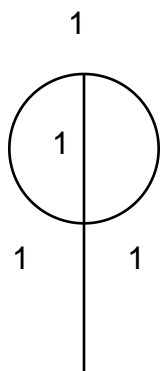


Figure 2 c

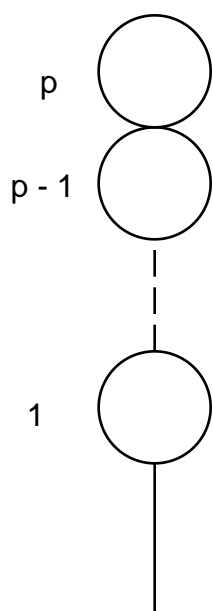


Figure 3

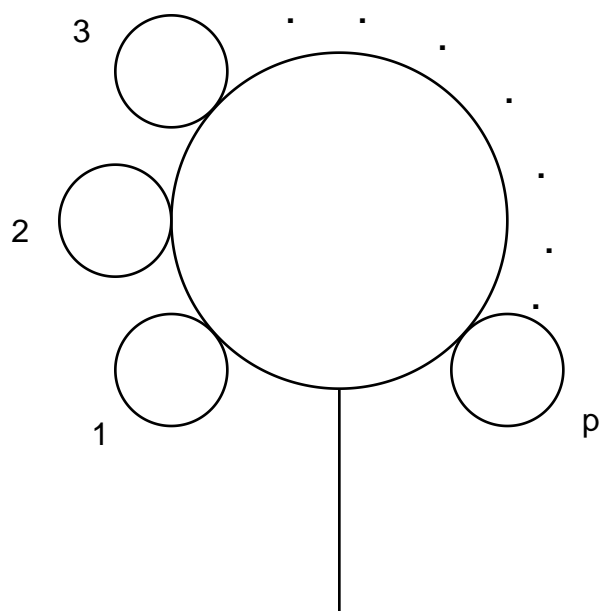


Figure 4

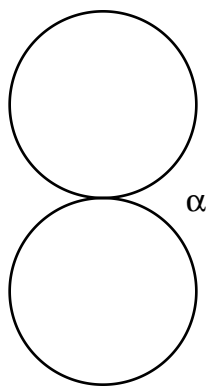


Figure 5